1. INTRODUCRION

The good comfort performance of the vehicle is a very important requirement for most of the vehicles. To evaluate this performance, however, the experimental on field is expensive and inconvenient. By mathematically model the vehicle, we can make this evaluation during the design phase of the manufacture, and it is also a much more convenient way. Lumped parameter models are commonly used for assessing performance of different dynamic system, especially in automotive research. The whole vehicle can be regarded as two parts: sprung mass and unsprung mass. The current study covers suspension system modeling using lumped parameter models of increasing degrees of complexity such as 2 degrees-of-freedom system (quarter-car model), four degrees-of freedom system (half-car model) and seven degrees-of-freedom system (full-car model).

In this paper we focus on 7 d.o.f. full car model. This model consists of a sprung mass and four unsprung mass represented by four wheels. Between sprung mass and unsprung masses are compliance represented by spring and damper. Between unsprung masses and the ground are just the stiffness of the tire ignoring the damping of the tire. The sprung mass has 3-degree of freedom representing body bounce (Zs), roll (ϕ) and pitch (θ) movement, while the unsprung masses has 4-degree of freedom in vertical motions (Zu_f1, Zu_f2, Zu_r1, Zu_r2). We choose a compact truck and find the relevant data in Trucksim. Based on these data, we build the 7 d.o.f. mathematical model and calculate the differential equations of each 7 d.o.f. variables. Then we output the bounce (Zs), roll (ϕ) and pitch (θ) motions of the vehicle body and compares the results with the plot from Trucksim to verify the accuracy of our mathematical model.

2. MATHEMATICAL MODEL

In this model, the vehicle aerodynamic effect is neglected and the road is assumed to be level except for road disturbance. Based on the 7-degree of freedom model in Fig. 1, the displacement of sprung mass can be calculated as (Where $\mathrm{m_s}$ is the sprung mass of the vehicle, $\ddot{\mathrm{z}}_\mathrm{s}$ is the acceleration of sprung mass. F represent the force. In the sub note, S for spring, D for damper, F for front, R for rear, L for left, R for right):

$$m_s\ddot{z}_s = -F_{SFL} - F_{DFL} - F_{SFR} - F_{DFR} - F_{SRL} - F_{DRL} - F_{SRR} - F_{DRR}$$

Acceleration at unsprung masses can be given by (Where m_u represent the unsprung mass and T for tire):

$$\begin{cases} m_{u\_fr}\cdot\ddot{z}_{u\_fr} = F_{SFR} + F_{DFR} - F_{TFR} \\ m_{u\_fl}\cdot\ddot{z}_{u\_fl} = F_{SFL} + F_{DFL} - F_{TFL} \\ m_{u\_rr}\cdot\ddot{z}_{u\_rr} = F_{SRR} + F_{DRR} - F_{TRR} \\ m_{u\_rl}\cdot\ddot{z}_{u\_rl} = F_{SRL} + F_{DRL} - F_{TRL} \end{cases}$$

The pitch motion of the sprung mass θ can be given by (where J_y is the moment of inertia about y axis, a is the distance between front axle and center of gravity, b is the distance between rear axle and center of gravity):

$$J_y\ddot{\theta} = -(F_{SFL} + F_{DFL} + F_{SFR} + F_{DFR}) \cdot a + (F_{SRL} + F_{DRL} + F_{SRR} + F_{DRR}) \cdot b$$

The roll motion of the sprung mass can be given by (where J_x is the moment of inertia about x axis and t represent the wheel track):

$$J_x\ddot{\varphi} = -(F_{SFL} + F_{DFL} + F_{SRL} + F_{DRL}) \cdot t + (F_{SFR} + F_{DFR} + F_{SRR} + F_{DRR}) \cdot t$$

Represent the all the forces with stiffness k and damping c, these seven equations can be given by:
Sprung mass:

$$m_s\ddot{z}_s = -k_{sf}(z_{s\_fl} - z_{u\_fl}) - c_{sf}(\ddot{z}_{s\_fl} - \ddot{z}_{u\_fl}) - k_{sf}(z_{s\_fr} - z_{u\_fr}) - c_{sf}(\ddot{z}_{s\_fr} - \ddot{z}_{u\_fr}) - k_{rf}(z_{s\_rl} - z_{u\_rl})$$
$$- c_{sr}(\ddot{z}_{s\_rl} - \ddot{z}_{u\_rl}) - k_{sr}(z_{s\_rr} - z_{u\_rr}) - c_{sr}(\ddot{z}_{s\_rr} - \ddot{z}_{u\_rr})$$

$$J_y\ddot{\theta} = -\left[k_{sf}(z_{s\_fl} - z_{u\_fl}) + c_{sf}(\dot{z}_{s\_fl} - \dot{z}_{u\_fl}) + k_{sf}(z_{s\_fr} - z_{u\_fr}) + c_{sf}(\dot{z}_{s\_fr} - \dot{z}_{u\_fr})\right] \cdot a$$
$$+ \left[k_{sr}(z_{s\_rl} - z_{u\_rl}) + c_{sr}(\dot{z}_{s\_rl} - \dot{z}_{u\_rl}) + k_{sr}(z_{s\_rr} - z_{u\_rr}) + c_{sr}(\dot{z}_{s\_rr} - \dot{z}_{u\_rr})\right] \cdot b$$

$$J_x\ddot{\varphi} = -\left[k_{sf}(z_{s\_fl} - z_{u\_fl}) + c_{sf}(\dot{z}_{s\_fl} - \dot{z}_{u\_fl}) + k_{sr}(z_{s\_rl} - z_{u\_rl}) + c_{sr}(\dot{z}_{s\_rl} - \dot{z}_{u\_rl})\right] \cdot t$$
$$+ \left[k_{sr}(z_{s\_fr} - z_{u\_fr}) + c_{sf}(\dot{z}_{s\_fr} - \dot{z}_{u\_fr}) + k_{sr}(z_{s\_rr} - z_{u\_rr}) + c_{sr}(\dot{z}_{s\_rr} - \dot{z}_{u\_rr})\right] \cdot t$$

Unsprung mass:

$$m_{u\_fr} \cdot \ddot{z}_{u\_fr} = k_{sf}(z_{s\_fr} - z_{u\_fr}) + c_{sf}(\dot{z}_{s\_fr} - \dot{z}_{u\_fr}) - k_t(z_{u\_fr} - h_{fr})$$
$$m_{u\_fl} \cdot \ddot{z}_{u\_fl} = k_{sf}(z_{s\_fl} - z_{u\_fl}) + c_{sf}(\dot{z}_{s\_fl} - \dot{z}_{u\_fl}) - k_t(z_{u\_fl} - h_{fl})$$
$$m_{u\_rr} \cdot \ddot{z}_{u\_rr} = k_{sr}(z_{s\_rr} - z_{u\_rr}) + c_{sr}(\dot{z}_{s\_rr} - \dot{z}_{u\_rr}) - k_t(z_{u\_rr} - h_{rr})$$
$$m_{u\_rl} \cdot \ddot{z}_{u\_rl} = k_{sf}(z_{s\_rl} - z_{u\_rl}) + c_{sr}(\dot{z}_{s\_rl} - \dot{z}_{u\_rl}) - k_t(z_{u\_rl} - h_{rl})$$

Rearranging these seven equations, the second order differential equations of this 7 d.o.f. model can be given as (Where $X = \{z_s \quad \theta \quad \varphi \quad z_{u\_fl} \quad z_{u\_fr} \quad z_{u\_rl} \quad z_{u\_rr}\}^T$):

$$M \cdot \ddot{X} + C \cdot \dot{X} + K \cdot X = F$$

In detail, M, C, K and F can be given as:

$$M = \begin{bmatrix} m_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{u\_fl} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{u\_fr} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{u\_rl} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_{u\_rr} \end{bmatrix}$$

$$K = \begin{bmatrix} 2(k_{sf}+k_{sr}) & 2(bk_{sr}-ak_{sf}) & 0 & -k_{sf} & -k_{sf} & -k_{sr} & -k_{sr} \\ 2(bk_{sr}-ak_{sf}) & 2(a^2k_{sf}+b^2k_{sr}) & 0 & ak_{sf} & ak_{sf} & -bk_{sr} & -bk_{sr} \\ 0 & 0 & 2(t^2k_{sf}+t^2k_{sr}) & -tk_{sf} & tk_{sf} & -tk_{sr} & tk_{sr} \\ -k_{sf} & ak_{sf} & -tk_{sf} & k_{sf}+k_t & 0 & 0 & 0 \\ -k_{sf} & ak_{sf} & tk_{sf} & 0 & k_{sf}+k_t & 0 & 0 \\ -k_{sr} & -bk_{sr} & -tk_{sr} & 0 & 0 & k_{sr}+k_t & 0 \\ -k_{sr} & -bk_{sr} & tk_{sr} & 0 & 0 & 0 & k_{sr}+k_t \end{bmatrix}$$

$$C = \begin{bmatrix} 2(c_{sf}+c_{sr}) & 2(bc_{sr}-ac_{sf}) & 0 & -c_{sf} & -c_{sf} & -c_{sr} & -c_{sr} \\ 2(bc_{sr}-ak_{sf}) & 2(a^2c_{sf}+b^2c_{sr}) & 0 & ac_{sf} & ac_{sf} & -bc_{sr} & -bc_{sr} \\ 0 & 0 & 2(t^2c_{sf}+t^2c_{sr}) & -tc_{sf} & tc_{sf} & -tc_{sr} & tc_{sr} \\ -c_{sf} & ac_{sf} & -tc_{sf} & c_{sf} & 0 & 0 & 0 \\ -c_{sf} & ac_{sf} & tc_{sf} & 0 & c_{sf} & 0 & 0 \\ -c_{sr} & -bc_{sr} & -tc_{sr} & 0 & 0 & c_{sr} & 0 \\ -c_{sr} & -bc_{sr} & tc_{sr} & 0 & 0 & 0 & c_{sr} \end{bmatrix}$$

$$F = \{0 \quad 0 \quad 0 \quad k_t h_{fl} \quad k_t h_{fr} \quad k_t h_{rl} \quad k_t h_{rr}\}^T$$

We will further use this model in Simulink to have the numerical results.

3. TRUCKSIM MODEL

3.1 TruckSim model of the vehicle

TruckSim is used to obtain the relevant parameters of this data and verify the accuracy of this mathematical model. The relevant parameters is based on a compact truck given by TruckSim. The detail of the data is shown in figures below.

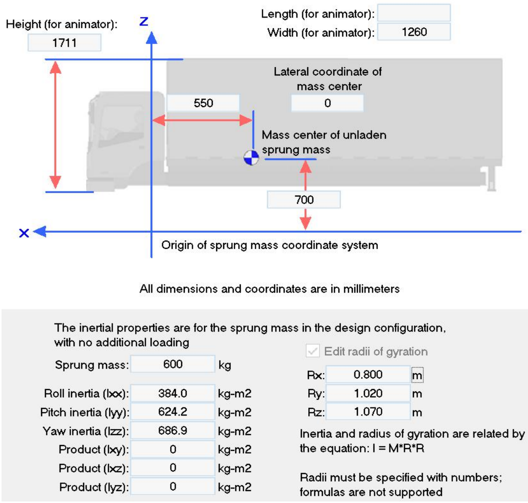

Figure 3.1 Sprung mass parameters

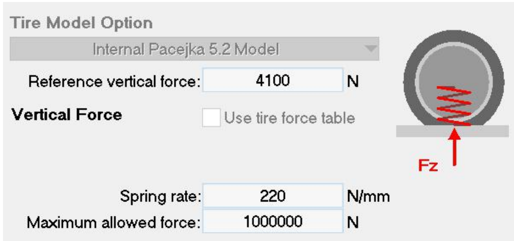

Figure 3.2 Unsprung mass parameters

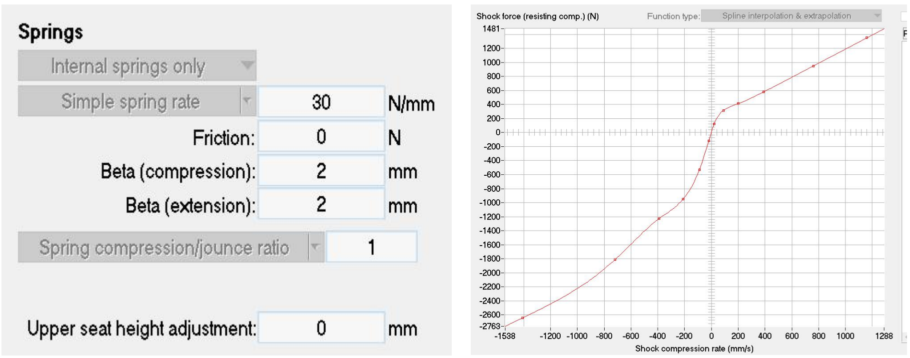

Figure 3.3 Front axle parameters

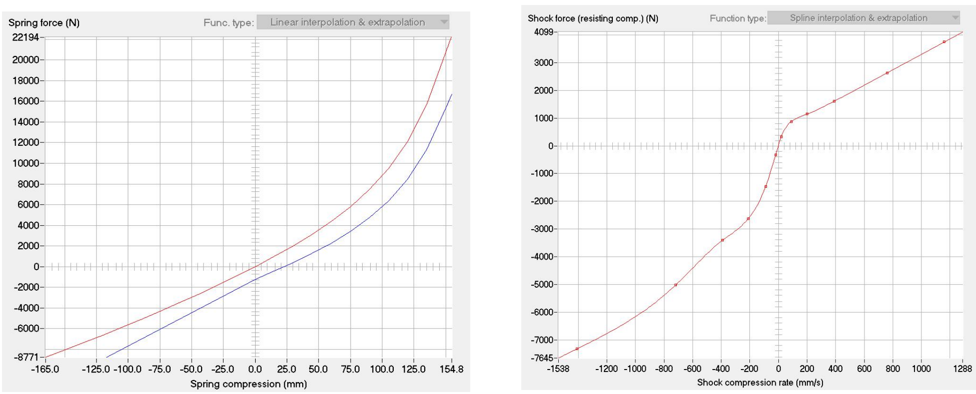

Figure 3.4 Rear axle parameters

All of the parameters we need are summerized in the Table 3.1

Table 3.1 Parameters of 7 d.o.f. model

Notation	Values	Units	Notation	Values	Units
m_s	600	kg	k_t	220000	N/m
J_y	624.2	Kg*m2	c_{sf}	5200	N*s/m
J_x	384	Kg*m2	c_{sr}	5200	N*s/m
m_u	105	kg	a	0.55	m
k_{sf}	15000	N/m	l	1.925	m
k_{sr}	31000	N/m	t	0.63	m

3.2 TruckSim model of the bumps

To use TruckSim to verify the results of Simulink obtained from the mathematical model, we let the vehicle running across the left/right bumps. The 3D and 2D profiles are shown in figure 3.5. During this procedure, the speed of the vehicle is 8 km/h. The vehicle start at station 100 m shown in the 2D profile and the bump starts at station 109 m and ends at station 121 m. This whole procedure takes about 14 seconds covering 30 m. The length and width of each bump is 6 m and the height of each bump is 0.1 m.

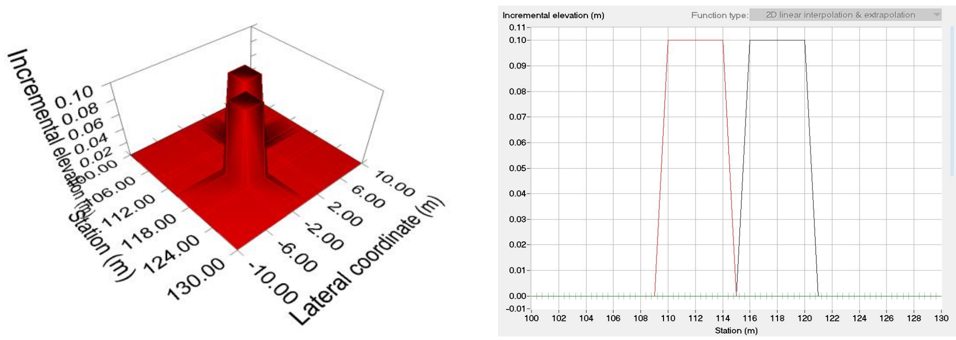

Figure 3.5 3D and 2D profiles of the bumps

4. SIMULATE THE MATHEMATICAL MODEL

Based on the second order differential equations, the Simulink block diagram model can be built shown in Figure 4.1. (we plus 0.05 to Z_s Trucksim data to offset the vertical displacement caused by the weight of the vehicle)

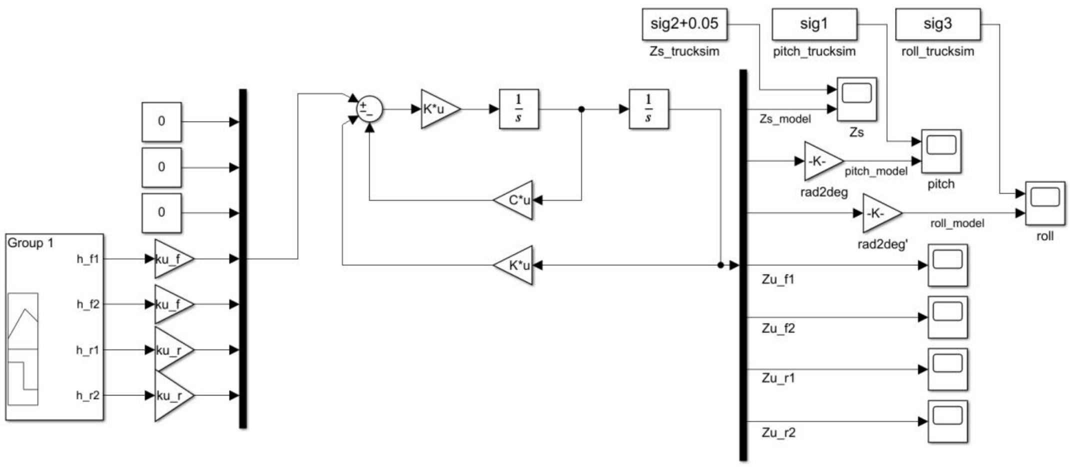

Figure 4.1 7 d.o.f. Simulink Model

In this Simulink model, the four input signals are h_{rr}, h_{rl}, h_{fl}, h_{fr}. The signal builder is used to draw the profiles of these four inputs. The profiles are shown in Figure 4.2.

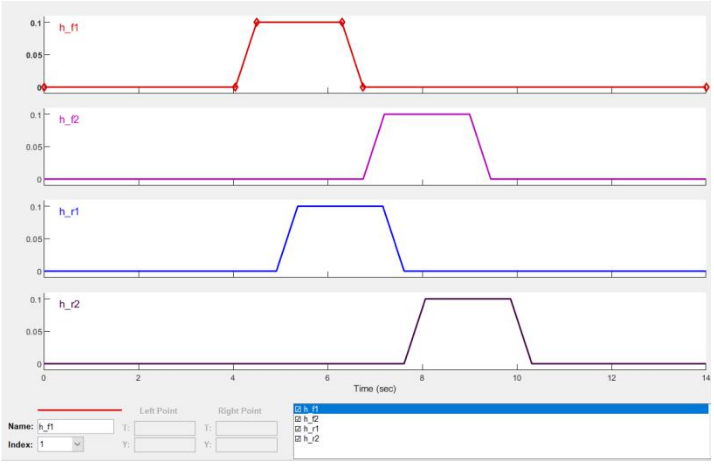

Figure 4.2 Profiles of the bumps in Simulink

5. Comparison of Simulink model and Trucksim model

Running the Simulink model and compares the result with data collected in Trucksim, we have these figures. Figure 5.1 and 5.2 show the Simulink and Trucksim simulation of roll (ϕ) and pitch (θ) performances (the unit is deg), which have the similar trend but slightly differences in magnitude. Figure 5.3 shows the simulation result of Simulink sprung mass vertical displacement (z_s) which has the same trend but some differences in magnitude (the unit is m). The error maybe due to simplified model used in Simulink, while Trucksim model that is based on actual tested vehicle simulation process thus becomes more precise.

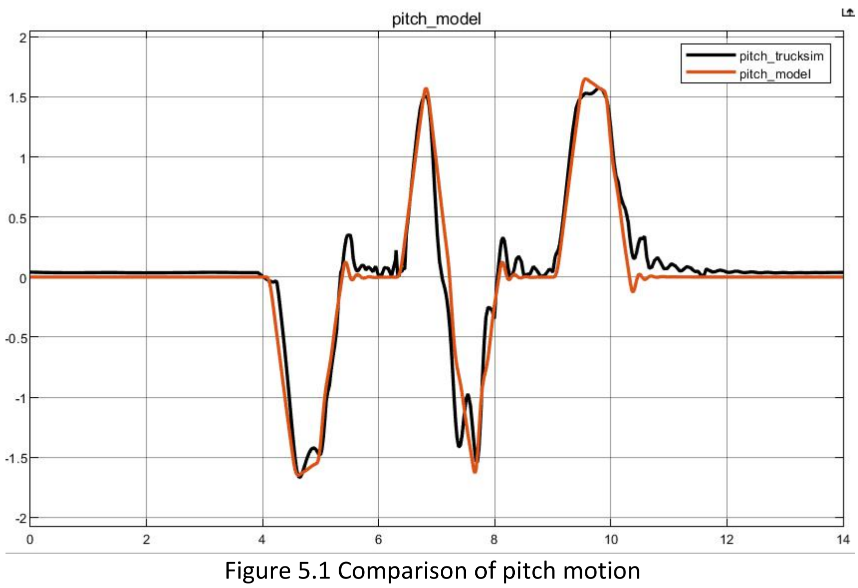

Figure 5.1 Comparison of pitch motion

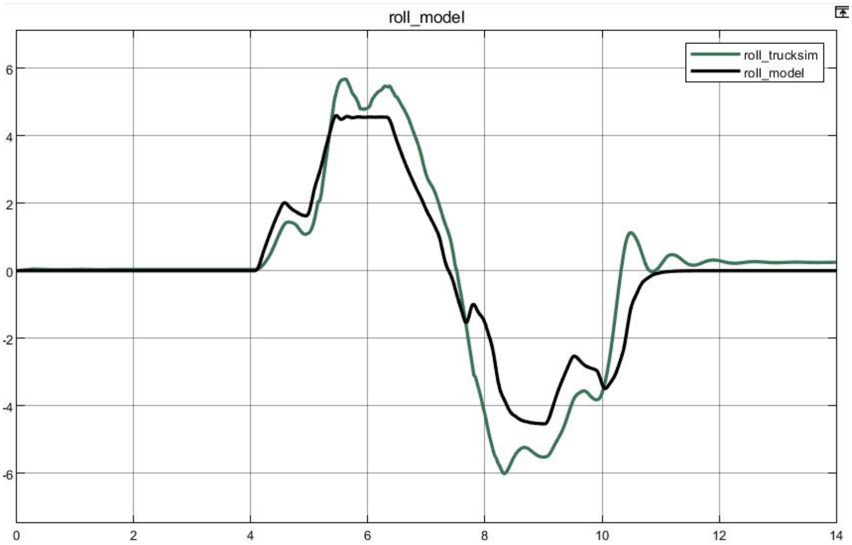

Figure 5.2 Comparison of roll motion

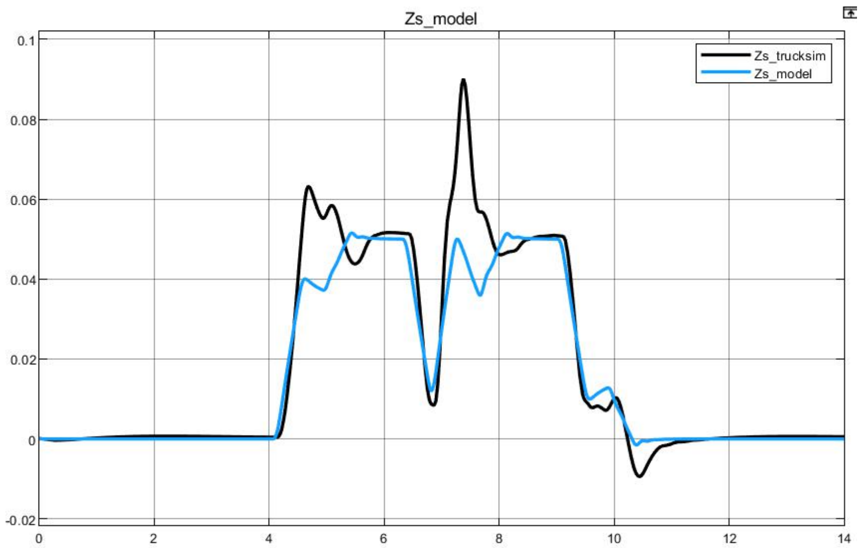

Figure 5.3 Comparison of sprung mass vertical displacement

In conclusion, since we made a lot of simplification when building this mathematical model, like we neglect the aerodynamic effect of the vehicle and so on, but we still get the similar result as Trucksim. So, we can basically ensure the accuracy of our mathematical model.